A Method for Deriving Mortality Estimates from Incomplete Vital Statistics

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INTRODUCTION

In countries in which population data are still defective, demographers have paid little attention to vital statistics.[1] Brass attributes this lack of interest to the absence of a theoretical model adapted to the analysis of these data.[2]

Yet, incomplete vital registration records may well yield a fairly accurate estimation of the main mortality indicators, such as life tables or crude death rates. A first attempt was made, some years ago, to improve the health statistics of Lebanon.[3] This method, which relied on the life tables of a certain number of countries was later complemented by using Princeton model life tables.[4] This method, which gave very similar results to those obtained by the previous one, was successfully applied to the data of two other countries, Jordan[5] and Libya[6]. In this paper we illustrate the different stages of this method by applying them to data relating to the Malagasy Republic.

In many developing countries efforts have recently been devoted to the implementation or improvement of the registration of vital events.[7] The construction of appropriate methods for processing such data is, therefore, of considerable importance. As no special surveys are required no additional costs are incurred. Moreover, the procedure has the advantage over other methods of adjustment that no assumptions are required relating to stability or quasi-stability of the population or constancy of mortality in the recent past.

The general principle of the method will be briefly explained before applying it to Malagasy data.

PRINCIPLES AND ASSUMPTIONS

Underregistration of deaths is common in vital registration systems in many developing countries. Moreover, early childhood deaths are less completely registered than those at older ages everywhere for when births escape registration subsequent deaths are likely to do so as well. Yet, above a certain age[8] when births are no longer registered, there is no reason why the completeness of death registration should depend on the age at death. Our first assumption is thus, that there exists an age beyond which the rate of underregistration of deaths does not vary significantly.[9] In other

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[1] A method for dealing with mortality was devised by Carrier, in 1958. ('A note on the estimation of mortality and other population characteristics, given deaths by age', *Population Studies* **12**, pp. 149–163). This method which assumes stability of the population studied may not given satisfactory results. For example, in Jordan (in 1951) the estimated crude death rate was 10.7 per cent, whereas it should have been close to, or even higher than, 20 per cent. See W. Brass, *Methods for Estimating Mortality from Limited and Defective Data* (Chapel Hill: The North Carolina Population Centre, 1975).

[2] Brass, *op. cit.* in footnote 1.

[3] Y. Courbage and Ph. Fargues, *Some Methodological Elements Proper to Lebanese Data in Order to Obtain Basic Indices on Mortality*, UNESOB/WHO. Expert Group Meeting on Mortality (Beirut, 1972).

[4] Y. Courbage and Ph. Fargues, *La situation démographique au Liban*, tome I (Beyrouth, 1973).

[5] Brass, *op. cit.* in footnote 1.

[6] Mahmoud S. Abdou Issa, 'Estimation of Mortality Level in Libya: 1972', in *Mortality Trends and Differentials in Some Arab and African Countries* (Cairo: Cairo Demographic Centre, 1975).

[7] In the francophone countries of Africa, for instance, OCAM (Organisation Commune Africaine et Mauricienne) and UDEA-Tchad (Union Douanière et Economique des Etats d'Afrique Centrale) have already established projects with the object of expanding and improving vital registration.

[8] This age could not exceed minimum school age.

[9] We cannot truthfully state that the probability of a death being recorded is statistically independent of age. The many reasons for non-registration are still insufficiently known to establish a relation with the age of the deceased. On the other hand, it is not certain, either, that the assumption is wrong. In our view the likelihood is

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words, above this age the age distribution of registered deaths differs only very slightly from the actual (but unknown) age distribution of deaths.

Moreover, any decrease in mortality in a population normally consists of a shift towards older ages of deaths previously occurring at younger ages. Thus, for a given age structure of the population, a fall in mortality leads to a concentration of deaths at older ages. For a given population age structure, the age structure of deaths and the level of mortality are related.

A second assumption, which will be discussed in detail later, is that the age structure of deaths of the country under study is similar to a set of standard age-specific death rates.[10]

These assumptions are later combined and provide the basis of the method: a knowledge of the distribution of deaths by age, and of a family of life tables to which this distribution could be related, makes it possible to deduce the true level of mortality (measured by the death rate above a given age) from the relation between age structure of deaths and level of mortality. This true level, is then compared with registered mortality, thus making possible an estimate of the extent of underregistration of deaths, and a later adjustment of observed age-specific death rates, as the overall underregistration rate applies to each individual age group.

BASIC DATA REQUIRED

Two series of data are required:

(1) Death statistics by age obtained either from vital registration records or from a census or survey in which questions on vital events taking place within the last twelve months are included.

(2) The age and sex distribution of the population.

Table 1. *Registered deaths in Madagascar (1965-67)*

Age group	1965		1966		1967		Average 1965–67	
	Males	Females	Males	Females	Males	Females	Males	Females
0	7,355	6,307	8,836	7,561	9,620	8,152	8,604	7,340
1–4	6,263	6,223	8,361	7,808	8,635	8,356	7,753	7,462
5-9	1,403	1,273	1,947	1,722	2,045	1,857	1,798	1,617
10-14	770	632	922	825	1,111	957	934	805
15-19	758	846	908	999	962	1,102	876	982
20-24	747	934	786	1,036	805	1,056	779	1,009
25-29	850	1,309	902	1,405	957	1,364	903	1,359
30-34	883	1,076	968	1,290	950	1,234	934	1,200
35-39	1,142	1,214	1,291	1,464	1,086	1,292	1,173	1,323
40–44	1,129	1,114	1,316	1,313	1,308	1,184	1,251	1,204
45–49	1,452	1,256	1,771	1,506	1,626	1,410	1,616	1,391
50-54	1,595	1,172	1,914	1,649	1,642	1,501	1,717	1,441
55-59	1,655	1,257	2,177	1,620	2,087	1,495	1,973	1,457
60–66	1,857	1,509	2,174	1,865	1,834	1,610	1,955	1,661
65–69	1,710	1,308	2,471	1,849	2,221	1,717	2,134	1,625
70+	4,812	4,601	6,452	6,504	5,523	5,610	5,596	5,572
Unknown*	431	338	352	300	282	224	355	287
Total	34,812	32,369	43,548	40,716	42,694	40,121	40,351	37,735

Source: Institut National de la Statistique et de la Recherche Economique, *Inventaire Socio-èconomique,* tome 1 (Tananarive, 1969).

* Deaths of unknown age were omitted in further calculations.

that the assumption holds: the pattern of the mortality rates thus obtained does not differ greatly from those of existing life tables.

[10] This condition is very flexible. It is sufficient to find a family of age-specific death rates in which the age distribution of deaths in the population studied in two large age groups (e.g. 1-50 years and 50 years and over, or 5-60 years and 60 years and over), for a given death rate above a certain age (e.g. one year or five years) is close to that which would have been observed if mortality had followed the selected pattern exactly.

Table 2. *Age and sex distribution of the population in Madagascar in 1966 (in thousands)*

Age group	Males	Females	Age group	Males	Females
0	133	132	40–44	133	141
1–4	432	430	45–49	125	127
5-9	484	456	50-54	106	98
10–14	428	387	55-59	84	89
15-19	269	301	60-64	60	54
20-24	193	227	65–69	51	46
25-29	163	230	70 +	76	72
30-34	156	179	N.D.	2	2
35-39	154	180	Total	3,049	3,151

Source: Institut National de la Statistique et de la Recherche Economique, *Enquête dèmographique, Madagascar, 1966* (Tananarive, 1967).

Registered deaths should be sufficiently numerous to enable their age distribution to be accurately estimated. As an example, in Madagascar statistics on deaths were available for the three calendar years 1965-67. Hence, the calculations refer to the average number of deaths during these three years rather than to those of 1966 only - the year of the survey for which the age structure of the population is available. This averaging tends to minimize chance fluctuations and to avoid errors caused by shifting deaths from one year to another.

The age distribution of the population should be given for the middle of the period for which death statistics are available. If it is not given, any reliable age distribution can be projected to the middle of that period. Moreover, age pyramids are often distorted because of age mis-statement. But, as mis-statements of age are likely to affect deaths as well as the enumerated population, it seems preferable not to adjust the age structure. But, if the final series of rates obtained from these calculations seems biased, an adjustment of the age structure may well become necessary.[11]

A DETAILED EXAMPLE OF THE METHOD

Notation:

Registered deaths	D'
Actual deaths	D
Population	P
Registered death rate:	m'
Actual death rate	m
Closed age group	$(x, x+a)$
Open age group	$(x+)$

Any age-specific death rate can be written:

$$m(x, x+a) = \frac{D(x, x+a)}{P(x, x+a)} = \frac{P(a+)}{P(x, x+a)} \cdot \frac{D(x, x+a)}{D(a+)} \cdot \frac{D(a+)}{P(a+)} \qquad (1)$$

Thus, the age-specific death rate is the product of three factors:

(1) The proportion in the age group $(x, x+a)$ of the population aged a and over $(x \geq a)$, where a

[11] In the case of Lebanon, as the age structure was obtained from a sample survey, such an adjustment was thought necessary.

is a young age used as a lower limit in subsequent calculations. This proportion is known since a reliable age-sex distribution of the population is available.

(2) The proportion of deaths at ages $(x, x + a)$ among total deaths aged a or over. This proportion can be estimated by the proportion:

$$\frac{D'(x, x+a)}{D'(a+)}$$

by virtue of the assumption of a constant rate of underregistration of deaths at ages above a.

(3) The death rate of the open age interval a and over. The method of estimating this ratio is shown below.

The rate $m(a+)$ is unknown since the registered rate $m'(a+)$ is obviously underestimated,[12] and is a measure of the level of mortality in this age group. For a given population age structure there is a relation between this mortality level and the age distribution of deaths beyond age a. This last quantity may be represented by a single index, which measures the concentration of deaths at older ages:

$$i(a, \beta) = \frac{D(\beta+)}{D(a+)}$$

where β is an advanced age used as the second limit in the subsequent calculations. According to our first assumption $i(a, \beta)$ can be estimated by the ratio:

$$\frac{D'(\beta+)}{D'(a+)}$$

In the case of Madagascar, two values have been used for each one of the two thresholds a and β: $a = 1$ and 5 years and $\beta = 50$ and 60 years. The are thus four possible values of $i(a, \beta)$.

It remains to find out which level of death rate at ages above a corresponds to a given index $i(a, \beta)$. Existing life tables will be used as a standard pattern for this estimation.

For each sex, separately, the age-specific death rates of several life tables are applied to the

Table 3. *Index of concentration* i(a, β) *of deaths at older ages: Madagascar (1965-67)*

$i(a, \beta)$	Males	Females
$i(1, 50)$	0.4261	0.3905
$i(1, 60)$	0.3085	0.2942
$i(5, 50)$	0.5658	0.5191
$i(5, 60)$	0.4097	0.3912

Source: Computed from Table 1.

[12] The values of the registered death rates in Madagascar for the open groups $a+$ are as follow (per thousand):

a	Males	Females
1 year	10.8	10.0
5 years	9.5	8.7

Table 4. *Values of* m(a +) *(per thousand) and* i(a, β) *associated with the age structure of the population in Madagascar and death rates of the Princeton life tables, Model 'West' (males)*

Level	5	6	7	8	9	10	11	12	13	14	15
$m(1+)$	26.52	24.42	22.50	20.74	19.12	17.63	16.23	14.94	13.60	12.51	11.46
$i(1.50)$	0.3563	0.3663	0.3772	0.3890	0.4018	0.4157	0.4310	0.4478	0.4700	0.4922	0.5183
$i(1,60)$	0.2552	0.2645	0.2746	0.2855	0.2974	0.3104	0.3246	0.3403	0.3607	0.3804	0.4043
$m(5+)$	22.40	20.84	19.40	18.09	16.87	15.73	14.68	13.70	12.73	11.97	11.16
$i(5,50)$	0.4952	0.5040	0.5134	0.5237	0.5348	0.5467	0.5596	0.5735	0.5894	0.6042	0.6252
$i(5,60)$	0.3546	0.3638	0.3737	0.3844	0.3959	0.4082	0.4215	0.4358	0.4523	0.4674	0.4876

Source: Calculations in the Appendix.

appropriate populations of Madagascar. These calculations give, for each life table, values $i(a, \beta)$ and $m(a\,+)$, so that a set of numerical correspondences between $i(a, \beta)$ and $m(a\,+)$ is obtained.

${}_aM_x$ is the age-specific death rate of age group $(x, x + a)$ in a life table, then, separately for each sex:

$$m(a\,+) = \sum_{x\,=\,a}^{\infty} P(x, x+a)\,M(x, x+a) \quad \sum_{x\,=\,a}^{\infty} P(x, x+a)$$

$$i(a, \beta) = \sum_{x\,=\,\beta}^{\infty} P(x, x+a)\,M(x, x+a) \quad \sum_{x\,=\,a}^{\infty} P(x, x+a)\,M(x, x+a)$$

The products ${}_aP_x \cdot {}_aM_x$ are the deaths which would have occurred in age group $(x, x + a)$ in Madagascar if the central age-specific death rates of the life table had applied. We have used the rates of the Princeton Model 'West' life tables from level 5 to 15. The results for men only are presented in Table 4. For women, exactly the same method was used.

The results of Table 4 can be represented graphically. Four nomograms are thus obtained showing a perfect one-one correspondence between the level measured by $m(a\,+)$, and the structure of mortality measured by $i(a, \beta)$. Since $i(a, \beta)$ can be directly deduced from the registered deaths of a given population by reading off the value on the nomogram, the death rate $m(a\,+)$ corresponding to the index of concentration of deaths at older ages in this population may be obtained.[13] The amount of underregistration of deaths above age a and the correction factor can then be deduced.

In the Madagascar example, this calculation was performed for each of the four values $i(a, \beta)$. Two of the correction factors relate to deaths over the age of one year, and two to deaths over the age of five years. However, the underregistration rate, and, therefore, the correction factor for a given sex, vary very little with the thresholds a and β. In our example, the correction factor does not change perceptibly, whether we take $a = 1$ or $a = 5$. It seems preferable, in this case, to choose $a = 1$ in order to apply the adjustment to all deaths recorded as taking place at ages above one year.[14]

If we had chosen $a = 5$ we would not have been able to adjust deaths in the age group 1–4. The choice of β, however, might affect the correction factor rather more. For $\beta = 50$, the underregistration rates come out a little lower than if $\beta = 60$, both for males and females. However, considering that age at death is more likely to be wrongly given for older persons, better results would be obtained for lower values of β; therefore β was taken as 50 rather than 60 years.[15]

Thus, for Madagascar, we can estimate $m(1\,+)$ from the value of $i(1, 50)$ derived from registered deaths. The correction factor finally obtained, is the ratio $m(1\,+)/m'(1\,+)$. For males this factor is 1.54 and for females 1.34.

The adjusted age-specific death rates can then be obtained by applying these correction factors to all age groups over one year:[16]

$$\text{Males } {}_am_x = 1.54\ {}_am'_x$$

$$\text{Females } {}_am_x = 1.34\ {}_am'_x$$

[13] The construction of a nomogram for eleven levels of life tables is presented in this paper only to illustrate the one-one correspondence between $m(a\,+)$ and $i(a, \beta)$. In practice, the two levels of life tables which enclose the values of $i(a, \beta)$ have to be found by trial and error and the value of $m(a\,+)$ obtained by interpolation.

[14] In the case of Lebanon, on the other hand, the choice $a = 5$ years seemed more appropriate.

[15] K. Vaidyanathan, who has applied this method in an unpublished paper, has chosen a value of β as low as 30 years, using $i(5, 30)$ in: K. Vaidyanathan, 'A Simple Method for Estimation of Death Rate and Expectation of Life at Birth from Defective Registration Statistics', quoted by M. S. A. Issa *op. cit.* in footnote 6, p. 16.

[16] The relation ${}_am_x = m(1\,+)/m'(1\,+)\ {}_am'_x$ is equivalent to Equation 1 in this section with $a = 1$

$$(1) \quad {}_am_x = \frac{P(1\,+)}{{}_aP_x} \times \frac{{}_aD_x}{D(1\,+)} \times \frac{D(1\,+)}{P(1\,+)} \quad x \geq 1 \text{ as } {}_aD_x/D(1\,+) = {}_aD'_x/D'(1\,+).$$

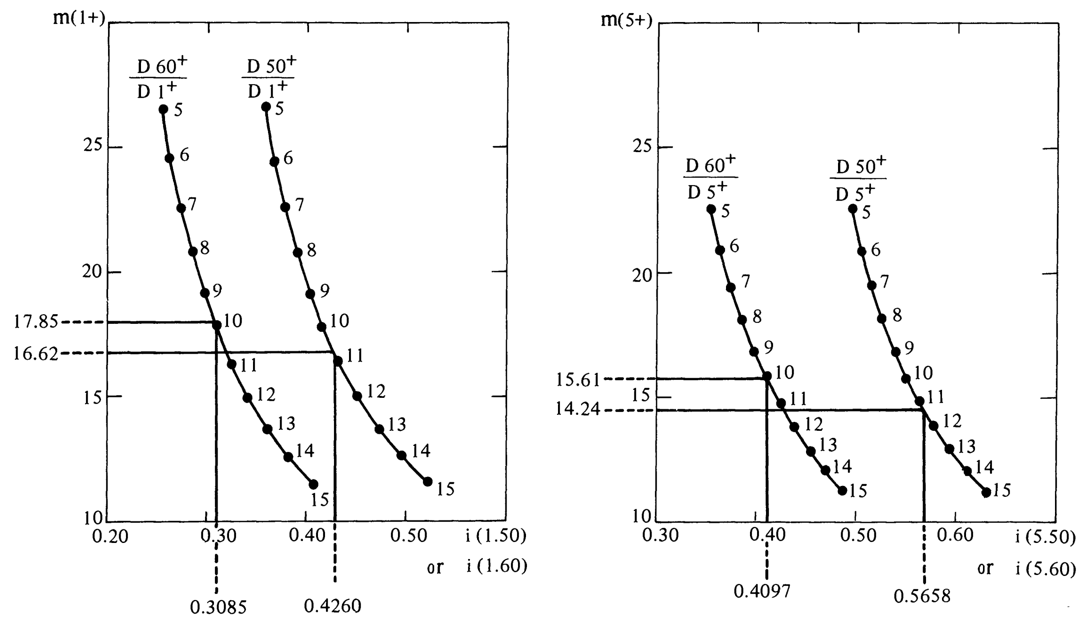

Figure 1. Nomogram showing the relation between the index of concentration of deaths at older ages $i(\alpha, \beta)$ and the overall death rate above age α: $m(\alpha+)$

Table 5. *Estimation of the level of mortality above age α and of the correction factor for registered deaths*

α (1)	β (2)	$i(\alpha, \beta)$ (3)	Death rate at ages above α: Adjusted $m(\alpha +)$ (4)	Registered $m'(\alpha +)$ (5)	Rate of underregistration $(6) = \frac{(4)-(5)}{(4)}$	Correction factor $(7) = \frac{(4)}{(5)}$
			MALES			
1	50	0.4260	16.62	10.78	35	1.54
1	60	0.3085	17.85	10.78	40	1.66
5	50	0.5658	14.24	9.52	33	1.50
5	60	0.4097	15.61	9.52	39	1.64
			FEMALES			
1	50	0.3905	13.38	9.98	25	1.34
1	60	0.2942	14.01	9.98	29	1.40
5	50	0.5191	11.51	8.75	24	1.32
5	60	0.3912	12.27	8.75	29	1.40

Source: Adjusted rates are obtained from Table 4, registered rates from basic data of Tables 1 and 2.

We may write (1) as

$${}_a m_x = \frac{P(1+)}{{}_aP_x} \times \frac{{}_aD'_x}{D'(1+)} \times \frac{D(1+)}{P(1+)} = {}_am'_x \times \{m(1+)/m'(1+)\}.$$

It should be noted that, for ages over one, the classical formula for the conversion of central death rates into probabilities of dying:

$$_nq_x = \frac{2n\ _nm_x}{2 + n\ _nm_x} \text{ has been used.}$$

However, the application of these correction factors to the registered death rates below one year of age yields results which are clearly too low:

Males $1.54 \times 64.69 = 99.62$

Females $1.34 \times 55.61 = 74.52$

These results are far below known levels of infant mortality in African countries which are close to or higher than 150. But, as infant deaths are likely to be less completely registered than deaths above one year of age, it seems preferable to deduce infant mortality rates from the Princeton life tables, considering the close connection between infant mortality rates and death rates in early childhood: hence $_1q_0$ is determined by interpolation between the two levels of the 'West' tables which enclose the value of $_4q_1$ found for Madagascar. Therefore, $_1q_0$ is the only rate which is directly derived from model life tables.[17]

MAJOR FINDINGS

These calculations lead to the construction of the life tables of Madagascar 1965-67 (Table 6). These life tables may be compared with the Princeton models. But these sets were used only as standards for the estimation of the overall death rate (one year and above). This means that they merely determined a level of mortality. But the structure of these life tables depends only on the Madagascar data and is not affected by the use of model life tables.

Mortality rates in Madagascar for both sexes fluctuate about an average level in the model life tables (Level 10 for males, 11 for females), without greatly deviating from it. (Figure 2). The whole set of rates is enclosed within a narrow range: Levels 7 and 13, depending on age. This shows that the mortality structure given by this method is independent of that of model life tables; also that it is likely to yield true results. Differences observed between the mortality rates in Madagascar and those of the average level of the model life tables do not follow a regular pattern. If this were not so, it could indicate that underregistration of deaths depended upon age. The diagrams would then have shown age groups where all Madagascar mortality rates were higher than those of the average level, and others where they all were lower. The non-systematic character of these deviations tends to prove that the assumption that underregistration of deaths was independent of age is justified.[18]

The same method was applied to each sex separately. The consistency of the results of the two series of rates is worth noting: a male excess mortality at all ages but less acute between 15 and 40 years of age, because of increased mortality of women in the reproductive age groups. The difference between the average level of mortality for the two sexes (Level 10 for males and 11 for females) is due mainly to the female excess mortality between 5 and 20 years of age in the Princeton model life tables (Level 10, Model 'West'); there is no reason why such a pattern should exist in

[17] Statistical publications frequently give infant deaths by day, week or month of age. When the structure of infant mortality in tropical regions is better known, the same method as that devised here for the study of mortality above one year could be used to estimate $_1q_0$. The basis of this estimation would be the following observation: as infant mortality decreases, deaths tend to be more concentrated in the first days of life. A positive correlation should thus exist between $_1q_0$ and an index of concentration of infant deaths in the last months of the first year. This index could be the ratio of deaths aged 3-11 months to the deaths aged 0-11 months.

[18] If, for example, underregistration of deaths were to increase with age, the mortality rates computed for Madagascar would all have been above the average level at younger ages and below it at older ages.

Table 6. *Abridged life tables by sex in Madagascar (1965-67)*

Age	Registered rates $_am'_x$	Adjusted rates $_am_x$	Mortality rates $_aq_x$	Survivors l_x	Deaths $_ad_x$	Stationary population $_aL_x$	Stationary population T_x	Life expectancy $\mathring{e}_x$
				A. MALES				
0	64.69	(220.18)†	(187.17)*	10,000	1,872	8,502	407,151	40.72
1	17.95	27.64	104.77	8,128	851	30,810	398,649	49.05
5	3.72	5.73	28.25	7,277	206	35,870	367,839	50.55
10	2.18	3.36	16.66	7,071	118	35,061	331,969	46.95
15	3.26	5.02	24.79	6,953	172	34,336	296,908	42.70
20	4.04	6.22	30.62	6,781	208	33,386	262,572	38.72
25	5.54	8.53	41.76	6,573	274	32,180	229,186	34.87
30	5.99	9.22	45.06	6,299	284	30,785	197,006	31.28
35	7.62	11.73	56.98	6,015	343	29,218	166,221	27.63
40	9.41	14.49	69.92	5,672	396	27,370	137,003	24.15
45	12.93	19.91	94.83	5,276	500	25,128	109,633	20.78
50	16.20	24.95	117.43	4,775	560	22,475	84,505	17.70
55	23.48	36.16	165.81	4,215	699	19,326	62,030	14.72
60	32.58	50.17	222.89	3,516	784	15,620	42,704	12.15
65	41.84	64.43	277.46	2,732	758	11,766	27,084	9.91
70	73.63	113.39	1,000.00	1,974	1,974	15,318	15,318	7.76‡
				B. FEMALES				
0	55.61	(157.40)†	(139.84)*	10,000	1,398	8,882	463,258	46.33
1	17.35	23.25	88.87	8,602	765	32,878	454,376	52.82
5	3.55	4.76	23.52	7,837	184	38,725	421,498	53.78
10	2.08	2.79	13.85	7,653	106	37,999	382,773	50.02
15	3.26	4.37	21.61	7,547	163	37,326	344,774	45.68
20	4.44	5.95	29.31	7,384	217	36,378	307,448	41.64
25	5.91	7.92	38.83	7,167	278	35,141	271,070	37.82
30	6.70	8.98	43.91	6,889	303	33,689	235,929	34.25
35	7.35	9.85	48.07	6,586	316	32,141	202,240	30.71
40	8.53	11.43	55.56	6,270	349	30,478	170,099	27.13
45	10.95	14.67	70.76	5,921	418	28,560	139,621	23.58
50	14.70	19.70	93.88	5,503	517	26,221	111,061	20.18
55	16.37	21.94	104.00	4,986	519	23,633	84,840	17.02
60	30.77	41.23	186.89	4,467	834	20,250	61,207	13.70
65	35.32	47.33	211.61	3,633	769	16,241	40,957	11.27
70	77.38	103.69	1,000.00	2,864	2,864	24,716	24,716	8.63‡

* Estimated by linear interpolation on tables 'West' from the value of $_4q_1$.

† Computed from the formula $m_0 = q_0/(L_0/l_0)$ where $L_0 = 0.2\, l_0 + 0.8 l_1$.

‡ Computed from Princeton life tables at a level corresponding to the level of e_1 (obtained by linear interpolation between two consecutive tables).

Madagascar. After 25 years of age, the male excess mortality in Madagascar is of the same order of magnitude as in Level 10 of the model life tables (Figure 3).

The calculation of the adjusted age-specific death rates also leads to an estimate of the true number of deaths, and thus to a determination of the overall rates of underregistration: about half of the males and over one-third of the female deaths are not registered. This method has, therefore, coped with defective vital statistics, yet the results obtained are fairly plausible (table 7).

Underregistration seems more pronounced for males than for females. This could well be true. Yet as the age of females at death may well be overstated at about age 50 (there is a marked tendency to overstate the age of highly fertile women), the index of concentration of deaths $i(1, 50)$ may consequently be overestimated for females, leading then to an underestimation of the level of mortality at ages above one year.

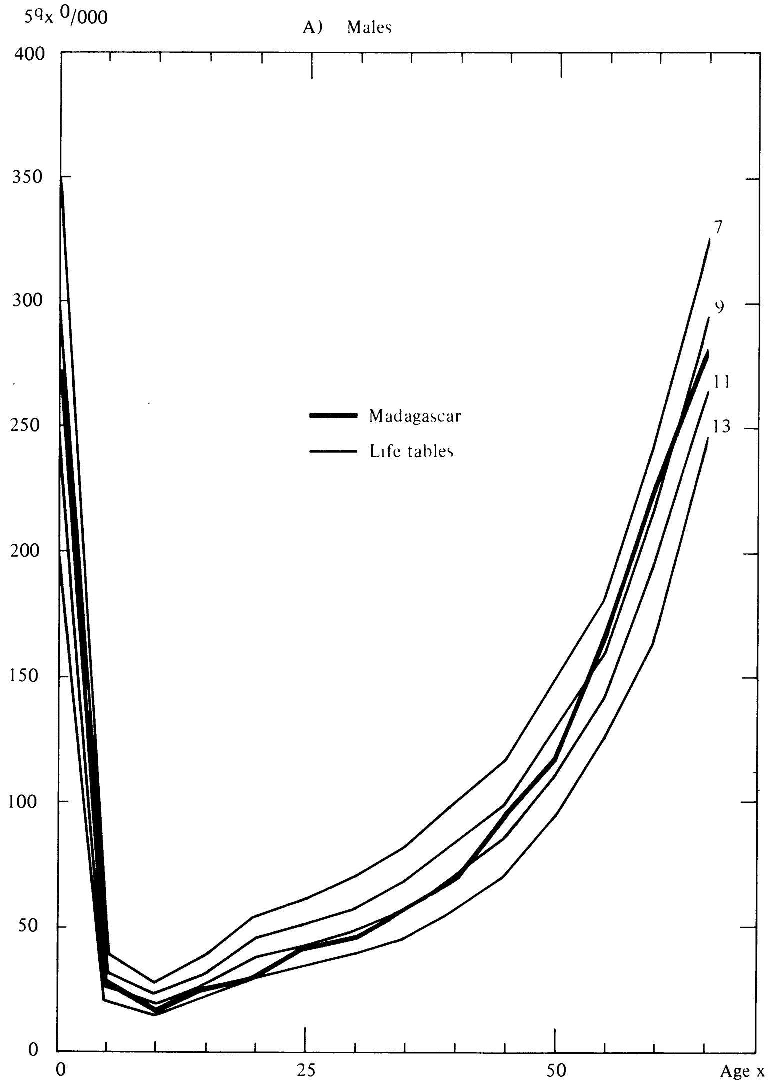

Figure 2A. Location of Madagascar Mortality Rates within the Network of Princeton Life Tables (Model 'West').

These results also make it possible to estimate the crude death rate in the country. In Madagascar in 1965-67 these rates were per thousand

Males: 25.5
Females: 19.4
All: 22.4

THE CASE OF INADEQUACY OF MODEL 'WEST'

As described previously, this method is valid wherever the age pattern of mortality in the country is fairly close to that of the model life tables selected as standard for the estimation of the death rate $m(a+)$.

The closeness of the two structures should, therefore, be checked. In the example of Madagascar, as shown in Figure 2, this relation seems quite satisfactory; the mortality rates in Madagascar deviate in a random manner around those of the 'West' table with the closest life-expectancy (Level 10 for males, 11 for females).

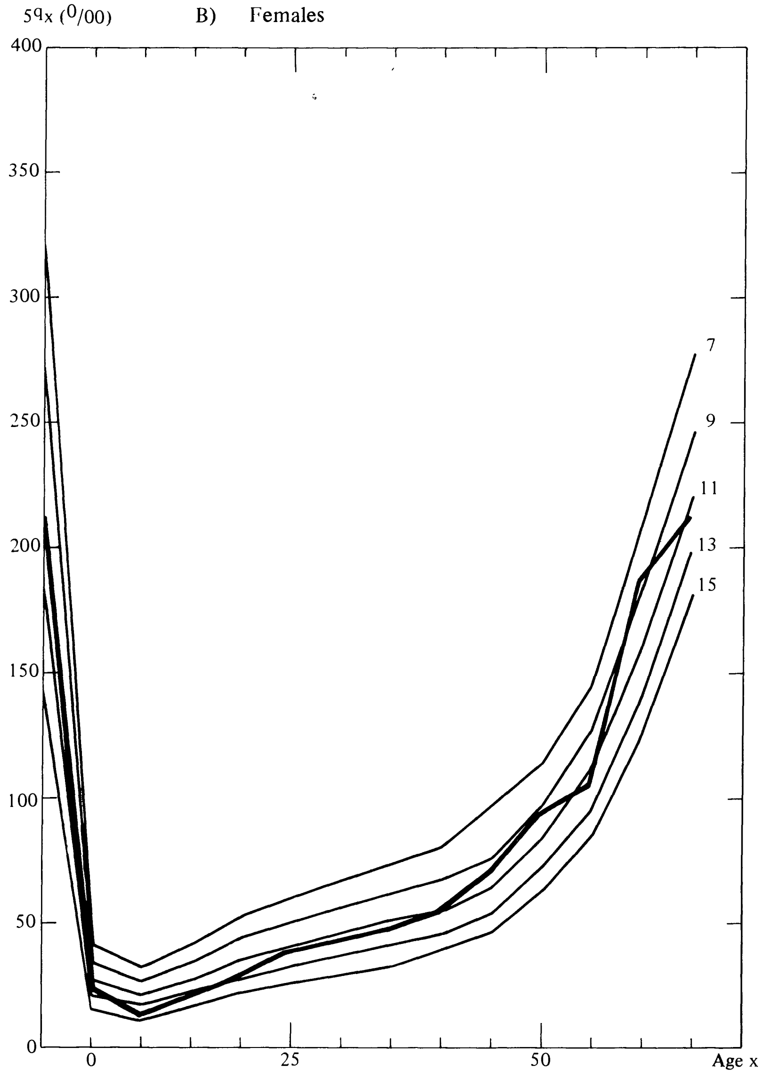

Figure 2B. Location of Madagascar Mortality Rates within the Network of Princeton Life Tables (Model 'West').

Contrariwise, in the case of Lebanon, the mortality rates obtained by using Model 'West' as a standard, deviate perceptibly from the patterns of this model. Mortality at older ages was that of a higher level in these model life tables than at younger ages. The selection of a more appropriate model was made by calculating comparative indices, which related the mortality rates in Lebanon to those of the model life tables with the same life expectancy in the four families (Table 8).

This trial and error approach is essential. The choice of Model 'West' for Madagascar only implied a *level* of mortality for the country. However, it yielded an estimate of the *age-structure* of this mortality, which was definitely independent of Model 'West' and based only upon the Madagascar data.

A comparison of the mortality rates, obtained for Lebanon by using Model 'West' as a standard, with those of the four Princeton families, showed that:

(1) For the same level of mortality (measured by the life expectancy at birth), mortality rates of Table 'West' were relatively higher at younger ages and lower at older ages than those of Lebanon;

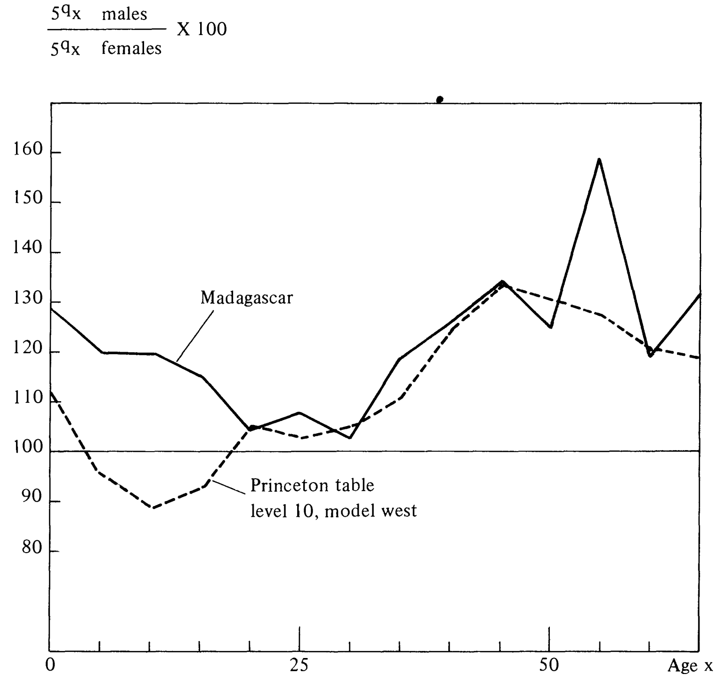

Figure 3. Excess male mortality indices.

Table 7. *Rates of underregistration of deaths*

Age	Registered deaths			Estimated deaths			Rate of underregistration (per cent)		
	Males	Females	All	Males	Females	All	Males	Females	All
0	8,604	7,340	15,944	29,284	20,776	50.060	71	65	68
1 and over	31,391	30,108	61,499	48,342	40,345	88,687	35	25	31
All ages	39,995	37,448	77,443	77,626	61,121	138,747	48	39	44

Source: Registered deaths are taken from Table 1, rate of underregistration and estimated deaths taken from Table 5.

(2) Models 'North' and 'East' were even further from Lebanese mortality than Model 'West';

(3) Model 'South' seemed closest to Lebanese mortality as regarded by the age structure of deaths above age 5. For the same life expectancy at birth, mortality was higher at ages below five in Table 'South' than in Lebanon and lower above this age. But the relative difference between the mortality in Model 'South' and that in Lebanon was about the same in the two age groups 5-59 years and 60 years and over. Therefore, we conclude that Lebanese mortality showed the same pattern of variation by age as Model 'South' above the age of five years.[19]

The calculations should therefore be started again (in the same order as above) using now the

[19] It is this closeness of the age structure of mortality in Lebanon above the age of five years to that of Model 'South' that justified the selection of $a = 5$ years in the case of Lebanon.

model closest to the age structure of deaths. In the case of Lebanon, the use of Model 'South' as standard for the estimation of mortality above five years of age has resulted in the mortality rates of Table 9. On Figure 4, it may be observed that the two curves of mortality rates obtained by using Models 'West' and 'South' respectively are parallel.[20] This proves once again that this method helps only to locate a level of mortality, but that the structure of mortality rates is independent of the selected model life tables.

Table 8. *Comparison of mortality rates in Lebanon with those of the model life tables*

Age	Lebanese mortality rates (computed with Model 'West' as standard)*	Mortality rates in the model life tables (‰)†				Comparative indices‡			
		'South'	'North'	'East'	'West'	'South'	'North'	'East'	'West'
0	57.97	81.88	53.33	71.77	56.52	1.412	0.920	1.238	0.975
1-4	20.98	30.86	28.32	18.44	20.35	1.471	1.350	0.879	0.970
5-9	8.76	6.37	13.00	6.58	7.51	0.727	1.484	0.751	0.857
10-14	6.33	4.52	8.05	4.74	5.81	0.714	1.272	0.749	0.918
15-19	7.52	6.64	11.27	7.95	9.22	0.883	1.499	1.057	1.226
20-24	10.14	9.35	15.54	11.22	12.73	0.922	1.533	1.107	1.255
25-29	11.14	10.27	16.77	12.00	14.85	0.922	1.505	1.077	1.333
30-34	13.95	12.23	18.34	13.57	15.56	0.877	1.315	0.973	1.115
35-39	17.01	14.44	20.91	16.81	19.70	0.849	1.229	0.988	1.158
40-44	22.29	19.37	26.03	22.22	25.61	0.869	1.168	0.997	1.149
45-49	30.59	26.34	31.94	31.84	35.30	0.861	1.044	1.041	1.154
50-54	48.14	39.76	45.25	47.88	50.88	0.826	0.940	0.995	1.057
55-59	69.85	56.44	60.14	71.39	74.04	0.808	0.861	1.022	1.060
60-64	117.53	87.32	90.97	108.25	111.07	0.743	0.774	0.921	0.945
65-69	177.99	136.87	139.90	167.49	166.42	0.769	0.786	0.941	0.935
70-74	248.99	224.84	215.87	261.94	252.23	0.903	0.867	1.052	1.013
75-79	367.91	366.07	326.34	394.40	373.43	0.995	0.887	1.072	1.015

Source: See Courbage and Fargues: *loc. cit.* in footnote 3, p. 17.

* This series of mortality rates has been obtained in using model 'West' as standard, provisionally.

† Mortality rates for both sexes obtained by using the mortality rates for each sex in the model life tables and the sex ratios by age of the Lebanese population, their level being that corresponding to $\mathring{e}_0 = 63.7$ years for both sexes (interpolated between two consecutive tables).

‡ Ratio of mortality rates in model life tables to those of Lebanon.

The reference to model life tables is perhaps not the best approach. Obviously, each set of model life tables summarizes the mortality experience of a large number of countries; thus, mortality in any one country may deviate, more or less, from all available models. So, the estimation of $m(a+)$ may well gain in precision if reference was to be made to the life tables of actual countries, rather than the Princeton models. Such sets of countries, should belong to a homogeneous geographical region containing the country studied. In Lebanon, we used this method as a first approach (Table 9 and Figures 4 and 5). It yielded results which were fairly close to those found subsequently by using model life tables.[21] Actually, the use of data from real countries is questionable, because little is known as a rule about mortality conditions in the region studied. This is why we consider that the reference to model life tables is at present, an adequate substitute, especially as the choice of the actual model has limited influence on the final results.

[20] Since ${}_1q_0$ and ${}_4q_1$ were estimated by another method, this parallelism is not observed at ages below five.

[21] The irregular deviations which appear on Figure 4 between the mortality rates obtained by this method and those obtained by using model life tables have come about because the age structure of the population was adjusted slightly in the second case but not in the first.

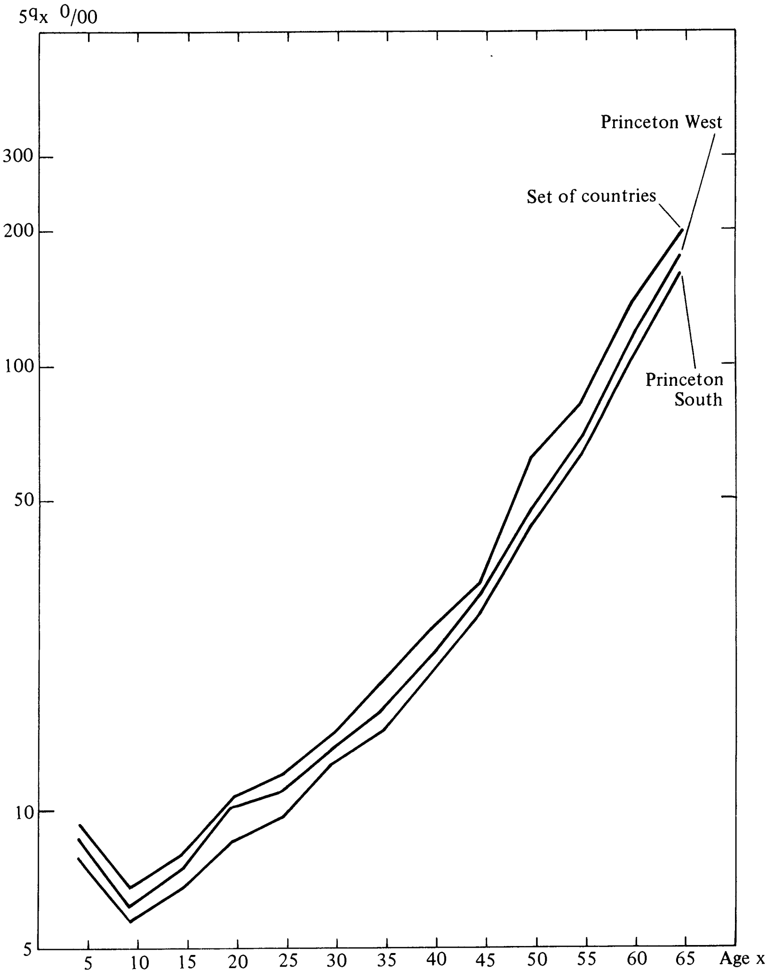

Figure 4. Mortality rates obtained with various family of life tables as standard, Lebanon, 1970.

Table 9. *Estimation of mortality rates in Lebanon (1970) using various families of tables as standard (both sexes)*

Age	Life tables selected as standard: Princeton 'West'	Princeton 'South'	Set of countries
0	57.97	65.10	64.50
1	20.98	22.23	22.51
5	8.76	7.92	9.45
10	6.33	5.73	6.88
15	7.52	6.78	8.07
20	10.14	8.76	10.89
25	11.14	9.85	12.11
30	13.95	12.94	14.86
35	17.01	14.97	19.01
40	22.29	20.70	25.94
45	30.59	27.12	32.16
50	48.14	43.75	63.86
55	69.85	62.92	81.38
60	117.53	104.13	142.09
65	177.99	163.84	208.25
e_0*	63.57	64.05	61.20

Source: As Table 8.
* Life expectancy at birth derived from the combination of these rates.

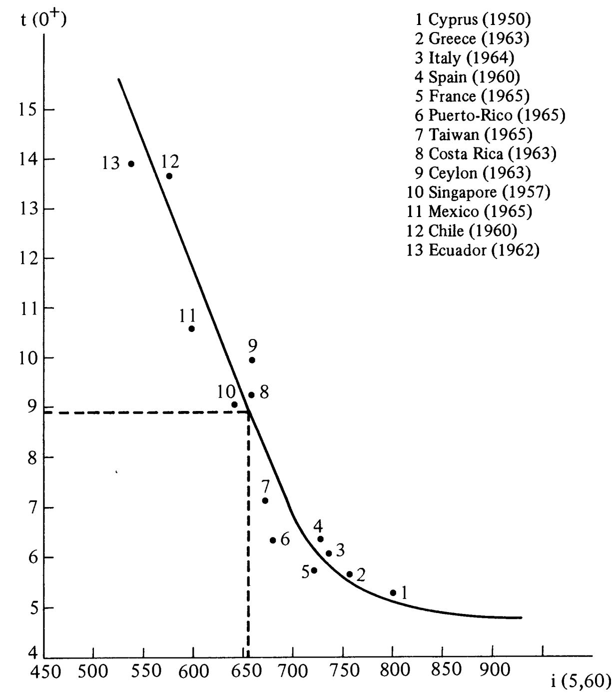

Figure 5. Mortality level and age distribution of deaths in selected countries: (1) Cyprus (1950); (2) Greece (1963); (3) Italy (1964); (4) Spain (1960); (5) France (1965); (6) Puerto-Rico (1965); (7) Taiwan (1965); (8) Costa Rica (1963); (9) Ceylon (1963); (10) Singapore (1957); (11) Mexico (1965); (12) Chile (1960); (13) Ecuador (1962).

APPENDIX

Calculation sheet of the death rate m(a +) and of the correction factor m(a +)/m'(a +)

Age $x, x+a$	Population (000) $_aP_x$	Number of average deaths 1965–67 $_aD_x$	Registered rates $_am'_x$	$_aP_x\ _am_x$ in different levels of Model 'West' tables 5	6	7	8	9	10	11	12	13	14	15	Interpolated rates	Correction factor $\frac{m(a+)}{m'(a+)}$
0	133	8,604	64.69	48.995	44,018	39,590	35,617	32,032	28,773	25,797	23,072	20,454	18,071	16,005		
1–4	432	7,753	17.95	21,669	19,423	17,392	15,548	13,859	12,308	10,873	9,547	8,027	6,756	5,707		
5–9	484	1,798	3.71	4,646	4,230	3,848	3,494	3,165	2,860	2,570	2,304	2,013	1,767	1,549		
10–14	428	934	2.18	2,936	2,679	2,444	2,221	2,016	1,823	1,644	1,477	1,288	1,138	1,006		
15–19	269	876	3.26	2,531	2,319	2,122	1,939	1,770	1,611	1,463	1,323	1,192	1,068	958		
20–24	193	779	4.04	2,600	2,378	2,175	1,986	1,812	1,648	1,496	1,353	1,220	1,092	979		
25–29	163	903	5.54	2,461	2,248	2,051	1,870	1,700	1,544	1,397	1,258	1,131	1,009	898		
30–34	156	934	5.99	2,733	2,493	2,271	2,069	1,880	1,703	1,540	1,387	1,245	1,262	983		
35–39	154	1,173	7.62	3,175	2,900	2,646	2,410	2,193	1,991	1,803	1,628	1,465	1,309	1,169		
40–44	133	1,251	9.41	3,332	3,047	2,785	2,546	2,324	2,117	1,925	1,745	1,576	1,420	1,282		
45–49	125	1,616	12.93	3,654	3,361	3,092	2,845	2,616	2,404	2,206	2,021	1,846	1,694	1,556		
50–54	106	1,717	16.20	3,954	3,653	3,377	3,124	2,891	2,674	2,473	2,285	2,101	1,953	1,818		
55–59	84	1,972	23.48	3,861	3,595	3,352	3,131	2,926	2,736	2,559	2,396	2,234	2,109	1,992		
60–64	60	1,955	32.58	3,801	3,553	3,328	3,121	2,932	2,756	2,594	2,444	2,298	2,185	2,080		
65–69	51	2,134	41.84	4,356	4,096	3,860	3,644	3,447	3,266	3,097	2,941	2,792	2,677	2,570		
70 +	76	5,596	73.63	11,562	11,167	10,812	10,489	10,194	9,920	9,667	9,429	9,203	9,023	8,852		
Total	3,047	39,995	13.13	126,266	115,165	105,151	96,054	87,757	80,134	73,104	66,610	60,085	54,533	49,404		
1 +	2,914	31,391	10.77	77,271	71,147	65,561	60,437	55,725	51,361	47,307	43,538	39,631	36,462	33,399		
5 +	2,482	23,638	9.52	55,602	51,719	48,163	44,889	41,866	39,053	36,434	33,991	31,604	29,706	27.692		
50 +	377	13,374	–	27,534	26,064	24,729	23,509	22,390	21,352	20,390	19,495	18,628	17,947	17,312		
60 +	187	9,685	–	19,719	18,816	18,000	17,254	16,573	15,942	15,358	14,814	14,293	13,885	13,502		
i(1, 50)	–	0.4260	–	0.3563	0.3663	0.3772	0.3890	0.4013	0.4157	0.4310	0.4478	0.4700	0.4922	0.5183	16.62	1, 54
i(5, 50)	–	0.5658	–	0.4952	0.5040	0.5134	0.5237	0.5348	0.5467	0.5596	0.5735	0.5894	0.6042	0.6252	14.24	1, 50
i(1, 60)	–	0.3085	–	0.2552	0.2645	0.2746	0.2855	0.2974	0.3104	0.3246	0.3403	0.3607	0.3808	0.4043	17.85	1, 66
i(5, 60)	–	0.4097	–	0.3546	0.3638	0.3737	0.3844	0.3959	0.4082	0.4215	0.4358	0.4523	0.4674	0.5876	15.61	1, 64
t(1 +)		10.78		26.52	24.42	22.50	20.74	19.12	17.63	16.23	14.94	13.60	12.51	11.46		
t(5 +)		9.52		22.40	20.84	19.40	18.09	16.87	15.73	14.68	13.70	12.73	11.97	11.16		